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Consequences of U dualities for Intersecting Branes in the Universe

S. Kalyana Rama

Institute of Mathematical Sciences, C. I. T. Campus,
Tharamani, CHENNAI 600 113, India.

email: krama@imsc.res.in

ABSTRACT

We consider N – charge, intersecting brane antibrane configurations in M theory which are smeared uniformly in the common transverse space and may describe our universe. We study the consequences of U dualities and find that they imply relations among the scale factors. We find using Einstein’s equations that U dualities also imply a relation among the density ρ and the pressure p_i for the single charge case. We present an ansatz for ρ and p_i for the N – charge case which yields all the U duality relations among the scale factors. We then study configurations with identical charges, and also with net charges vanishing. We find among other things that, independent of the details of the brane antibrane dynamics, such four charge configurations lead asymptotically to an effective $(3 + 1)$ – dimensional expanding universe.

1. Introduction

It is important to understand how our $(3 + 1)$ – dimensional universe may be described within string theory or, equivalently, within M theory. Enormous amount of work has been done addressing this issue, a sample of which is given in [1] – [9].

Chowdhury and Mathur proposed recently that mutually BPS, multi charge, intersecting brane antibrane configurations in M theory, smeared uniformly in the common transverse space, may describe our universe [5]. This is because the branes, and similarly antibranes, in such configurations form bound states, become fractional, support very low energy excitations, and thus have high entropy. In this paper, therefore, we consider such N – charge configurations with three or more common transverse directions.

We take the brane directions to be toroidal and study the consequences of U dualities for such configurations. U dualities here refer to suitable combinations of dimensional reduction, dimensional uplifting, S and T dualities. We find that U dualities imply relations among the scale factors, which then are characterised by N independent functions.

The energy momentum tensor, $T^\mu{}_\nu = \text{diag}(-\rho, p_i)$, for such a configuration may be determined, in principle, by brane antibrane dynamics. We find using Einstein's equations that, as a consequence of U duality, ρ and p_i for the single charge case obey a relation. We then present an ansatz for ρ and p_i for the N – charge case which yields all the U duality relations among the scale factors.

Using this ansatz, we study configurations with identical charges and find, among other things, that four charge configurations lead asymptotically to an effective $(3 + 1)$ – dimensional expanding universe. This result follows as a consequence of U dualities in M theory and is independent of the details of the brane antibrane dynamics.

We also study configurations with identical charges and with net charges all vanishing which, for entropic reasons, are likely to dominate the universe. Assuming a particular equation of state, we list the asymptotic solutions for a few such N – charge configurations and discuss some of their properties. We find that four charge configurations are likely to dominate the universe and may provide a detailed realisation of the maximum entropic principle that we have proposed recently in [9] to determine the number $(3 + 1)$ of large spacetime dimensions. Even otherwise, these configurations provide, at the least, a model for our $(3 + 1)$ – dimensional expanding universe.

This paper is organised as follows. We discuss the consequences of U duality for scale factors in section 2, and for energy momentum tensor in section 3. We then study configurations with identical charges in section 4, and those with net charges also vanishing in section 5. We present a brief summary and then conclude by mentioning a few issues for further study in section 6.

2. Consequences of U dualities for scale factors

Mutually BPS, non extremal, intersecting branes in M theory can be thought of as intersecting brane antibrane configurations [10], and describe black holes when localised in the common transverse space. A configuration with N – types of branes ¹ will be referred to as N – charge configurations. These branes intersect as per the rules given in [11, 12], form bound states, become fractional, support very low energy excitations, and thus have high entropy. Chowdhury and Mathur proposed recently that such intersecting configurations, when smeared uniformly in the common transverse space, may describe our universe [5]. In the following, we consider only such configurations with three or more common transverse directions.

N – charge intersecting configurations in M theory can be transformed into each other by suitable combinations of dimensional reduction, dimensional uplifting, S and T dualities, collectively referred to in the following as U dualities. Let \downarrow_i and \uparrow_i denote dimensional reduction and uplifting along i^{th} direction between M theory and type IIA string theory; T_j denote T duality along j^{th} direction in type IIA/B string theories; and S denote S duality in type IIB string theory. Then the U duality $\uparrow_i T_j S T_j \downarrow_i$ interchanges i and j . The U dualities of the type $\uparrow_i T_j T_k \downarrow_i$ transform one N – charge configuration to another.

For example, the U duality $\uparrow_3 T_4 T_5 \downarrow_3$ transforms the $N = 1$ configuration $2 : 12$ to $5 : 12345$, whereas $\uparrow_3 T_1 T_2 \downarrow_3$ transforms it to $W : 3$; and, $\uparrow_5 T_1 T_2 \downarrow_5$ transforms the $N = 4$ configuration $2255 : (12, 34, 13567, 24567)$ to $W555 : (5, 12345, 23567, 14567)$. ²

In this paper, we assume that N – charge intersecting brane antibrane configurations are smeared uniformly in the common transverse space and describe our universe. The spatial directions parallel to the branes are taken to be toroidal. The common transverse directions may also be taken to be toroidal, and with sufficiently large radii so as to describe our universe, or may simply be taken to be non compact. The corresponding line element ds is given by

$$ds^2 = -e^{2\lambda_0} d\tilde{t}^2 + \sum_i e^{2\lambda_i} dx_i^2 \quad (1)$$

¹Here and in the following, we use the terms ‘branes’, ‘brane antibrane’, ‘branes and antibranes’, et cetera interchangeably and also use them to mean ‘waves’, ‘wave antiwave’, ‘waves and antiwaves’, et cetera. A wave antiwave configuration is that obtained, for example, from M2 brane antibrane configuration by an appropriate U duality. The configurations considered in this paper always consist of branes and antibranes and/or waves and antiwaves. Hence, our interchangeable use of the terms above is unlikely to cause any confusion; also their intended meaning will be clear from the context.

²Our notation for the brane configurations is as follows: In configuration 2255, there are two types of M2 branes along the directions (x^1, x^2) and (x^3, x^4) , and two types of M5 branes along $(x^1, x^3, x^5, x^6, x^7)$ and $(x^2, x^4, x^5, x^6, x^7)$; and similarly for the corresponding antibranes. In W555, there are three types of M5 branes along the directions indicated and also a wave along x^5 . Similarly for other configurations.

where $i = 1, 2, \dots, 10$ and λ_i depend only on time.³ Note that the physical time t is given by $dt = e^{\lambda_0} d\tilde{t}$ and that one may set $\lambda_0 = 0$ with no loss of generality. Hence, in the following, we will not keep track of λ_0 and its transformations under U dualities.

We now study the implications of U dualities of the type $\uparrow_i T_j T_k \downarrow_i$ which transform N – charge configurations into each other. Under such a U duality, it can be shown that λ_i in equation (1) transform to λ'_i given by

$$\begin{aligned} \lambda'_i &= \lambda_i - 2\lambda \quad , \quad \lambda'_j = \lambda_k - 2\lambda \quad , \quad \lambda'_k = \lambda_j - 2\lambda \\ \lambda'_l &= \lambda_l + \lambda \quad , \quad l \neq i, j, k \quad ; \quad \lambda \equiv \frac{\lambda_i + \lambda_j + \lambda_k}{3} \quad . \end{aligned} \quad (2)$$

Consider, as an example, the configurations $2 : 12$ with scale factors e^{λ_i} , and $5 : 12345$ with scale factors $e^{\lambda'_i}$ and $W : 3$ with scale factors $e^{\lambda''_i}$ obtained by the U dualities $\uparrow_3 T_4 T_5 \downarrow_3$ and $\uparrow_3 T_1 T_2 \downarrow_3$ on $2 : 12$. The λ_i , λ'_i , and λ''_i obey the obvious symmetry relations⁴

$$2 : \lambda_1 = \lambda_2 \quad , \quad \lambda_3 = \dots = \lambda_{10} \quad (3)$$

$$5 : \lambda'_1 = \dots = \lambda'_5 \quad , \quad \lambda'_6 = \dots = \lambda'_{10} \quad (4)$$

$$W : \lambda''_3 \quad , \quad \lambda''_1 = \lambda''_2 = \lambda''_4 = \dots = \lambda''_{10} \quad . \quad (5)$$

The λ'_i obtained from equations (2), with $(i, j, k) = (3, 4, 5)$ and hence $\lambda = \lambda_{10}$, are

$$\begin{aligned} \lambda'_1 &= \lambda'_2 = \lambda_1 + \lambda_{10} \quad , \quad \lambda'_3 = \lambda'_4 = \lambda'_5 = -\lambda_{10} \\ \lambda'_6 &= \dots = \lambda'_{10} = 2\lambda_{10} \quad . \end{aligned} \quad (6)$$

The obvious symmetry relation $\lambda'_1 = \lambda'_3$, and the above equations, imply further relations among λ_i , and among λ'_i . Similarly for λ''_i also. The extra relations for λ_i , λ'_i , and λ''_i , which thus follow from U duality, are

$$\lambda_1 + 2\lambda_{10} = 0 \quad , \quad 2\lambda'_3 + \lambda'_{10} = 0 \quad , \quad \lambda''_{10} = 0 \quad . \quad (7)$$

Consequences of U duality can be similarly obtained for intersecting configurations also by this method: First consider a pair of configurations, with scale factors e^{λ_i} and $e^{\lambda'_i}$, which are related by U duality; write down the obvious symmetry relations for λ_i and λ'_i which may be obtained by inspection or by applying U dualities $\uparrow_i T_j S T_j \downarrow_i$ with suitable i, j ; the U duality

³If there is a wave along x then dx in equation (1) stands for $dx - A_\mu dx^\mu$ where A_μ is to be obtained from $A_{\mu\nu\rho}$ by the corresponding U duality. Such gauge fields will be incorporated in the energy momentum tensor, which will be given later by an ansatz. Explicit expressions for gauge fields are not needed then and, hence, will not be shown here although it is an interesting problem to obtain the gauge fields corresponding to the given ansatz.

⁴The obvious symmetry relations can also be obtained as consequences of U dualities $\uparrow_i T_j S T_j \downarrow_i$ for suitable i and j under which i and j are interchanged: $\lambda'_i = \lambda_j$, $\lambda'_j = \lambda_i$, $\lambda'_l = \lambda_l$, $l \neq i, j$.

equations (2) relate λ'_i to λ_i ; these relations, and the obvious symmetry relations for λ'_i , then imply further relations among λ'_i , and among λ_i , as illustrated in the example above. These extra relations are the consequences of U duality. We will see shortly the sense in which all the relations among λ_i – the obvious symmetry relations as well as the U duality ones – given in this paper may be taken to be satisfied.

This method is simple and yet powerful. It yields all the relations among λ_i and shows, as may be expected, that λ_i for N – charge configurations are characterised by N independent functions. For example, consider the configuration $W555 : (5, 12345, 23567, 14567)$ with scale factors e^{λ_i} obtained by the U duality $\uparrow_5 T_1 T_2 \downarrow_5$ on $2255 : (12, 34, 13567, 24567)$ with scale factors e^{λ_i} . The obvious symmetry relations for λ_i and λ'_i are

$$2255 : \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5 = \lambda_6 = \lambda_7, \lambda_8 = \lambda_9 = \lambda_{10} \quad (8)$$

$$W555 : \lambda'_1 = \lambda'_4, \lambda'_2 = \lambda'_3, \lambda'_5, \lambda'_6 = \lambda'_7, \lambda'_8 = \lambda'_9 = \lambda'_{10} \quad (9)$$

Expressing λ'_i in terms of λ_i using equations (2), and enforcing the obvious symmetry relations $\lambda'_1 = \lambda'_4$ and $\lambda'_2 = \lambda'_3$, then yields the U duality relations

$$\lambda_1 + \lambda_4 + \lambda_5 = \lambda_2 + \lambda_3 + \lambda_5 = 0, \quad \lambda'_1 + \lambda'_2 + \lambda'_6 = 0 \quad (10)$$

which also show that λ_i , and similarly λ'_i , can be characterised by four independent functions. Similar relations for other configurations can also be obtained straightforwardly by this method, but will not be presented here since they can all be obtained from equation (13) and an ansatz for energy momentum tensor, given below.

3. Consequences of U dualities for energy momentum tensor

The energy momentum tensor for the N – charge intersecting brane anti-brane configurations, which are smeared uniformly in the common transverse space, is of the form $T^\mu{}_\nu = \text{diag}(-\rho, p_i)$. Then, for the metric given by equation (1), Einstein's equations $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = T_{\mu\nu}$ in natural units with $8\pi G = 1$ become

$$\ddot{\Lambda}^2 - \sum_i \dot{\lambda}_i^2 = 2\rho, \quad \ddot{\lambda}_i + \dot{\lambda}_i \dot{\Lambda} - \ddot{\Lambda} - \dot{\Lambda}^2 = -\rho + p_i, \quad \Lambda = \sum_i \lambda_i \quad (11)$$

where $i = 1, 2, \dots, 10$ and overdots denote derivatives with respect to the physical time t . It follows from the above equations that

$$\ddot{\lambda}_i + \dot{\lambda}_i \dot{\Lambda} = p_i + \frac{\rho - P}{9} \equiv f_i, \quad P = \sum_i p_i \quad (12)$$

Note that if λ_i satisfy a relation $\sum_i k_i \lambda_i = 0$, where k_i are constants, then ρ and p_i must be such that the functions f_i defined above satisfy $\sum_i k_i f_i = 0$.

Conversely, if ρ and p_i are such that $\sum_i k_i f_i = 0$ then $\sum_i k_i (\ddot{\lambda}_i + \dot{\lambda}_i \dot{\Lambda}) = 0$. It then follows that $\sum_i k_i \dot{\lambda}_i = K e^{-\Lambda}$ where K is an integration constant. **(i)** If e^Λ grows sufficiently fast in the limit $t \rightarrow \infty$, namely if $e^\Lambda \simeq t^\alpha$ with $\alpha > 1$, then we have $\sum_i k_i \lambda_i \simeq \text{constant}$ in this limit, independently of the initial conditions. If $\alpha \leq 1$ ⁵ then $\sum_i k_i \lambda_i$ is a function of t in the limit $t \rightarrow \infty$ also, and depends on initial conditions. **(ii)** In such cases, we impose the condition $\sum_i k_i \lambda_i = 0$ at some initial time. Then $K = 0$, and we have $\sum_i k_i \lambda_i = \text{constant}$ for all time t . With no loss of generality, this *constant* can be set to zero by coordinate rescaling. It is in the sense **(i)** or **(ii)** that $\sum_i k_i f_i = 0$ implies $\sum_i k_i \lambda_i = 0$.

Thus, all the relations among λ_i – the obvious symmetry relations as well as the U duality ones – given in this paper may be taken to be satisfied in the sense **(i)** or **(ii)**, explained above. Which one applies to which configuration can be decided by assuming the relations $\sum_i k_i f_i = 0$, and finding α from the solutions in the limit $t \rightarrow \infty$.⁶

Consider the configuration 2 : 12. It is natural to assume that $p_1 = p_2 \equiv p_\parallel$ and $p_3 = \dots = p_{10} \equiv p_\perp$. Then, f_i obey the relations $f_1 = f_2$ and $f_3 = \dots = f_{10}$. Further, let f_i obey the relation $f_1 + 2f_{10} = 0$ also so that λ_i may satisfy, in the sense explained above, the obvious symmetry relations and the U duality one given in equations (3) and (7). Using the definition of f_i , it follows easily that $\rho + p_\parallel = 2p_\perp$. A similar analysis for 5 branes and waves then shows that the U duality relations in (7) imply that the corresponding ρ and p_i obey a relation which we write as

$$p_\parallel = z (\rho - p_\perp) + p_\perp \quad (13)$$

where \parallel or \perp denotes directions parallel or transverse to branes/wave, and $z = -1$ for 2 branes and 5 branes and $= +1$ for waves.

Equation (13), which is obtained as a consequence of U dualities in M theory, is one of the main results of this paper. It determines p_\parallel in terms of ρ and p_\perp which are, in general, functions of the brane and antibrane charges q and \bar{q} .⁷ These functions $\rho(q, \bar{q})$ and $p_\perp(q, \bar{q})$ may be determined, in principle, by brane antibrane dynamics. $\rho(q, \bar{q})$ and $p_\perp(q, \bar{q})$ must be same for 2 branes, 5 branes, and waves as follows from U dualities; but p_\parallel will be different and is given by equation (13). If $q = \bar{q}$, *i.e.* if the net charge vanishes, then p_\perp and p_\parallel may be thought of as functions of ρ .

⁵For Kasner type vacuum solutions, $\alpha = 1$.

⁶Obtaining the solutions in the general case is difficult. For a class of N charge solutions given in section 5, see also [5, 6], it turns out that $\alpha \geq 1$ always, and $\alpha > 1$ for $N > 1$ and $\alpha = 1$ for $N = 1$. If we assume that this is the generic behaviour in general then **(i)** applies for $N > 1$ cases, and **(ii)** for $N = 1$ cases.

⁷The brane charge $q \propto n\tau V$ where n , τ , and V denote the number, tension, and volume of the branes. Similarly for antibranes. For waves, τV is to be replaced by $\frac{1}{R}$ where R is the size of the wave direction. Also, note that if the common transverse space is compact then the net charges must vanish as follows from Gauss's law. Hence, we implicitly assume the common transverse space to be non compact in the general case where the net charges do not vanish.

Consider now N – charge intersecting configurations with $N > 1$. We assume that the corresponding energy momentum tensor is given, just as in the black hole case [12], by the ansatz

$$T^\mu{}_\nu = \sum_{I=1}^N T^\mu{}_{\nu(I)} \implies \rho = \sum_{I=1}^N \rho_{(I)} , \quad p_i = \sum_{I=1}^N p_{i(I)} \quad (14)$$

where $T^\mu{}_{\nu(I)} = \text{diag}(-\rho_{(I)}, p_{i(I)})$ is the energy momentum tensor of the I^{th} type of branes/wave, and $p_{i(I)}$ are given in terms of $p_{\parallel(I)}$ and $p_{\perp(I)}$ which obey the relation (13) with $z = z_{(I)} = -1$ for branes and $= +1$ for waves.⁸

It is straightforward to show that the above ansatz for (ρ, p_i) implies the obvious symmetry relations and, since $(\rho_{(I)}, p_{i(I)})$ obey equation (13) with $z = z_{(I)}$, also the U duality relations among λ_i . For example, (ρ, p_i) obtained from equation (14) for the configuration 2255 : (12, 34, 13567, 24567) are

$$\begin{aligned} \rho &= \rho_{(1)} + \rho_{(2)} + \rho_{(3)} + \rho_{(4)} \\ p_1 &= p_{\parallel(1)} + p_{\perp(2)} + p_{\parallel(3)} + p_{\perp(4)} \\ p_2 &= p_{\parallel(1)} + p_{\perp(2)} + p_{\perp(3)} + p_{\parallel(4)} \\ p_3 &= p_{\perp(1)} + p_{\parallel(2)} + p_{\parallel(3)} + p_{\perp(4)} \\ p_4 &= p_{\perp(1)} + p_{\parallel(2)} + p_{\perp(3)} + p_{\parallel(4)} \\ p_5 = p_6 = p_7 &= p_{\perp(1)} + p_{\perp(2)} + p_{\parallel(3)} + p_{\parallel(4)} \\ p_8 = p_9 = p_{10} &= p_{\perp(1)} + p_{\perp(2)} + p_{\perp(3)} + p_{\perp(4)} \end{aligned} \quad (15)$$

where $p_{\parallel(I)}$ is given by equation (13) with $z_{(I)} = -1$ for $I = 1, 2, 3, 4$. It follows after some algebra that f_i , given by equation (12), obey the relations

$$\begin{aligned} f_5 = f_6 = f_7 , \quad f_8 = f_9 = f_{10} \\ f_1 + f_4 + f_5 = f_2 + f_3 + f_5 = 0 \end{aligned} \quad (16)$$

which in turn imply, in the sense explained below equation (12), the obvious symmetry relations and the U duality ones for λ_i given in equations (8) and (10). We have similarly verified for various intersecting configurations that the relations among λ_i implied by equation (13) and the ansatz in equation (14) are the same as those obtained directly by U dualities.

4. Configurations with identical charges

Let 10^{th} direction be transverse to all of the N types of branes. Then $p_{10} = \sum_I p_{\perp(I)}$ and $p_i - p_{10} = \sum_I (p_{i(I)} - p_{\perp(I)})$. Note that if i^{th} direction is transverse to I^{th} type of brane then $p_{i(I)} = p_{\perp(I)}$. Hence, the sum in the

⁸The relation (13) among $\rho_{(I)}$, $p_{\parallel(I)}$, and $p_{\perp(I)}$ may also be taken as part of the ansatz, so that the $N > 1$ case is completely independent of the $N = 1$ case. Then, the relations among λ_i are likely to be satisfied in the sense (i) explained earlier, see footnote 6, namely in the limit $t \rightarrow \infty$, and independently of the initial conditions.

expression for $p_i - p_{10}$ is only over the remaining I, denoted as $I \ni i$, for which $p_{i(I)} = p_{\parallel(I)}$. Using equation (13) for $p_{\parallel(I)}$, it now follows that

$$p_i - p_{10} = \sum_{I \ni i} z_{(I)} \left(\rho_{(I)} - p_{\perp(I)} \right) . \quad (17)$$

Consider now configurations with identical brane and antibrane charges, namely with $q_1 = \dots = q_N$ and $\bar{q}_1 = \dots = \bar{q}_N$. Then, $\rho_{(1)} = \dots = \rho_{(N)}$ and $p_{\perp(1)} = \dots = p_{\perp(N)}$. Therefore, $\frac{\rho_{(I)} - p_{\perp(I)}}{\rho - p_{10}} = \frac{1}{N}$ and it follows from equation (17) that the ratios $\frac{p_i - p_{10}}{\rho - p_{10}}$ are constants, say z_i , and are given by

$$z_i = \frac{p_i - p_{10}}{\rho - p_{10}} = \sum_{I \ni i} \frac{z_{(I)}}{N} . \quad (18)$$

Since $z_{(I)} = -1$ for branes and $+1$ for waves, we have that, in N - charge configurations with identical charges, each type of branes wrapping i^{th} direction contributes $-\frac{1}{N}$ to z_i , whereas a wave in that direction if present contributes $+\frac{1}{N}$ to z_i ; the net z_i is then the sum of all these contributions. $z_i = 0$ if there are no branes or wave along i^{th} direction.

From the definition of z_i , it follows that $\rho - p_i = (1 - z_i)(\rho - p_{10})$. Using this in equation (11) for λ_i , and after some algebra, it follows that

$$\ddot{\Lambda} + \dot{\Lambda}^2 = \left(\frac{10 - \sum_j z_j}{9} \right) (\rho - p_{10}) \quad (19)$$

$$\ddot{\lambda}_i + \dot{\lambda}_i \dot{\Lambda} = l_i \left(\ddot{\Lambda} + \dot{\Lambda}^2 \right) , \quad l_i = 1 - \frac{9(1 - z_i)}{10 - \sum_j z_j} . \quad (20)$$

Since z_i and, hence, l_i are constants, equation (20) implies that $\lambda_i = l_i \Lambda$ in the sense explained below equation (12), *i.e.* in the limit $e^\Lambda \rightarrow \infty$, equivalently $t \rightarrow \infty$, and upto coordinate rescaling. For a given configuration with identical charges, the constants l_i can be calculated using equations (18) and (20). It can be verified that l_i thus obtained are the same as those obtained by applying U dualities directly to configurations with identical charges. Note here that the obvious symmetry relations are enhanced when charges are identical.

Note that the constants (z_i, l_i) and the relation $\lambda_i = l_i \Lambda$ depend only on brane and antibrane charges being identical; in particular, they are independent of the details of the functions $\rho(q, \bar{q})$ and $p_{\perp}(q, \bar{q})$ and, hence, of the details of the brane antibrane dynamics that determines them.

The most interesting case is the four charge configuration 2255 or W555 with identical charges, for which $z_1 = \dots = z_7 = -\frac{2}{4}$, $z_8 = z_9 = z_{10} = 0$, $l_1 = \dots = l_7 = 0$, and $l_8 = l_9 = l_{10} = \frac{1}{3}$. Hence, the sizes of the brane directions (x^1, \dots, x^7) become constant in the limit $t \rightarrow \infty$. It thus follows that the configuration 2255 or W555 with identical charges leads asymptotically to an effective $(3 + 1)$ - dimensional expanding universe. Note that this result follows as a consequence of U dualities in M theory and, as explained above,

is independent of the details of the brane antibrane dynamics that determines the functions $\rho(q, \bar{q})$ and $p_{\perp}(q, \bar{q})$.

Similarly, for the three charge configuration 222 or 2W5 with identical charges, $z_1 = \dots = z_6 = -\frac{1}{3}$, $z_7 = \dots = z_{10} = 0$, $l_1 = \dots = l_6 = 0$, and $l_8 = \dots = l_{10} = \frac{1}{4}$ leading, as before, to an effective $(4 + 1)$ – dimensional expanding universe.

5. Configurations with vanishing net charges

It is known that, for a given energy, the entropy of the intersecting brane antibrane configurations, which are localised in the common transverse space and describe black holes, is maximum when the charges are identical and the net charges all vanish, *i.e.* $q_1 = \dots = q_N = \bar{q}_1 = \dots = \bar{q}_N$ [5, 10, 13]. We assume this to be the case also for the intersecting configurations which are smeared uniformly in the common transverse space. Hence, in the following, we focus on such configurations since, for entropic reasons, they are likely to dominate the universe.

If the net charges all vanish, *i.e.* if $q_I = \bar{q}_I$ for $I = 1, \dots, N$, then p_{\perp} may be thought of as a function of ρ . We assume that $p_{\perp} = w\rho$ where w is a constant in the range $-1 \leq w \leq 1$. Then, equation (13) becomes

$$p_{\parallel} = (z(1 - w) + w) \rho . \quad (21)$$

The energy momentum tensor for the N – charge configuration is given by the ansatz in equation (14) and depends on w and $\rho_{(1)}, \dots, \rho_{(N)}$. If the charges are also identical then we have $q_1 = \dots = q_N = \bar{q}_1 = \dots = \bar{q}_N$. Equivalently $\rho_{(1)} = \dots = \rho_{(N)}$ and $p_{\perp(1)} = \dots = p_{\perp(N)}$. The pressure p_i is then given by

$$p_i = w_i \rho , \quad w_i = z_i (1 - w) + w \quad (22)$$

where the constants z_i are given by equation (18). The energy momentum tensor now depends on w and ρ only.

General solutions to the equations of motion (11) in this case can be obtained from those for anisotropic universe given recently by Chowdhury and Mathur in [5]. In this context, note that Chowdhury and Mathur also derive the energy momentum tensor for brane antibrane configurations in a certain approximation and obtain the corresponding w_i , which can also be obtained from the above expressions by setting $w = 0$.

Here, we present the solutions for a few N – charge configurations in the asymptotic limit $t \rightarrow \infty$. See [5, 6] for details. The functions f_i in equation (12), see also equation (20), are now given by

$$f_i = C_i \rho , \quad C_i = w_i + \frac{1 - \sum_j w_j}{9} = \left(z_i + \frac{1 - \sum_j z_j}{9} \right) (1 - w) . \quad (23)$$

If $\sum_i(1 - w_i)C_i > 0$ then the solutions in the limit $t \rightarrow \infty$ are independent of initial conditions and are given by

$$e^{\lambda_i} \simeq t^{\alpha_i} \quad , \quad \alpha_i = \frac{2C_i}{\sum_j(1 + w_j)C_j} \quad . \quad (24)$$

The exponents α_i thus describe the scale factors in the asymptotic limit $t \rightarrow \infty$. These exponents α_i , and also z_i which describe the equation of state (22), can be obtained straightforwardly in terms of w using equations (18), (23), and (24) for any N – charge configurations for which charges are identical and net charges vanish, *i.e.* for which $q_1 = \cdots = q_N = \bar{q}_1 = \cdots = \bar{q}_N$, and for which $\sum_i(1 - w_i)C_i > 0$.⁹ In Table I we list $(-Nz_i)$ and α_i for a few N – charge intersecting configurations. The coordinates are arranged so that $\alpha_1 \leq \alpha_2 \leq \cdots \leq \alpha_{10}$.

⁹ $\sum_i(1 - w_i)C_i = 0$ for $N = 1$. Equation (24) is still applicable if one assumes, as seems physically reasonable, that this 0 is approached from above [6].

	$-N z_i$	α_i
2255 555W	2, 2, 2, 2, 2, 2, 2, 0, 0, 0	$(0, 0, 0, 0, 0, 0, 0, 2, 2, 2) \frac{1}{3(1+w)}$
222 25W	1, 1, 1, 1, 1, 1, 0, 0, 0, 0	$(0, 0, 0, 0, 0, 0, 1, 1, 1, 1) \frac{1}{2(1+w)}$
225 55W	2, 2, 1, 1, 1, 1, 1, 0, 0, 0	$(-2, -2, 1, 1, 1, 1, 1, 4, 4, 4) \frac{1}{7+6w}$
255	2, 2, 2, 2, 2, 1, 1, 0, 0, 0	$(-1, -1, -1, -1, -1, 2, 2, 5, 5, 5) \frac{1}{2(4+3w)}$
555	3, 2, 2, 2, 2, 2, 2, 0, 0, 0	$(-1, 0, 0, 0, 0, 0, 0, 2, 2, 2) \frac{1}{3+2w}$
2W	1, 0, 0, 0, 0, 0, 0, 0, 0, 0	$(-2, 1, 1, 1, 1, 1, 1, 1, 1, 1) \frac{1}{4+3w}$
22 5W	1, 1, 1, 1, 0, 0, 0, 0, 0, 0	$(-1, -1, -1, -1, 2, 2, 2, 2, 2, 2) \frac{1}{5+3w}$
55	2, 2, 2, 1, 1, 1, 1, 0, 0, 0	$(-2, -2, -2, 1, 1, 1, 1, 4, 4, 4) \frac{1}{7+3w}$
25	2, 1, 1, 1, 1, 1, 0, 0, 0, 0	$(-1, 0, 0, 0, 0, 0, 1, 1, 1, 1) \frac{1}{2+w}$

Table I : z_i and α_i , calculated using equations (18), (23), and (24), for a few N – charge configurations with $q_1 = \dots = q_N = \bar{q}_1 = \dots = \bar{q}_N$. The corresponding w_i are given by equation (22). The scale factors $e^{\lambda_i} \simeq t^{\alpha_i}$ in the asymptotic limit $t \rightarrow \infty$.

We now make a few remarks.

(i) For the configurations 2255 and W555 with $m = 3$ common transverse directions, and for the $N = 3$ configurations 222 and 25W with $m = 4$ common transverse directions, the brane directions which are taken to be toroidal remain constant in size in the asymptotic limit $t \rightarrow \infty$ and the m common transverse directions expand. The scale factors for the transverse directions are identical to those of a $(m + 1)$ – dimensional expanding isotropic universe

containing a perfect fluid with the equation of state $p = w\rho$. This result follows as a consequence of U dualities in M theory, which was conjectured in [6] following different considerations. Thus, in particular, the 4 – charge configuration 2255 or W555 leads asymptotically to an effective $(3 + 1)$ – dimensional expanding isotropic universe with the six compact directions remaining constant in size.

(ii) Other three charge configurations in Table I can all be transformed to the configurations 222 or 25W by repeated applications of U dualities. The two charge configurations can also be transformed similarly to the configuration $2W : (12, 2)$ which, in turn, can be transformed to string theory configuration $FW : (2, 2)$, namely fundamental strings along x^2 direction with waves, by dimensional reduction along x^1 direction. The corresponding string metric and the dilaton ϕ can be obtained easily in the asymptotic limit $t \rightarrow \infty$ and are given by

$$ds^2 \simeq -d\tilde{t}^2 + \sum_{i=2}^{10} dx_i^2 \quad , \quad e^\phi \simeq \tilde{t}^{-\frac{1}{1+w}} \quad (25)$$

where $\tilde{t} \simeq t^{\frac{3(1+w)}{4+3w}}$. In the asymptotic limit, $\tilde{t} \rightarrow \infty$, the scale factors in string metric become constant, and the effective string coupling $e^\phi \rightarrow 0$.

(iii) Thus, for the above configurations, namely 2255, W555, 222, 25W in M theory and FW in string theory, the scale factors and, hence, the physical sizes of the compact directions remain constant in the asymptotic limit $t \rightarrow \infty$. We assume that these asymptotic constant sizes, which depend on initial conditions, are suitably large so that higher order corrections to the original action are negligible and, consequently, the present results remain valid.

In contrast, for the remaining configurations in Table I, one or more $\alpha_i < 0$ for the compact directions and, hence, the corresponding sizes $\rightarrow 0$ in the asymptotic limit $t \rightarrow \infty$ independent of the initial conditions. New light modes are then likely to appear in such cases, higher order corrections to the original action are likely to be non negligible [7, 8] and, consequently, the present results for such configurations are likely to be modified.

The correct description in such cases may be obtained, as explained in detail in [8], by using S, T, U dualities and transforming these configurations to the ‘safe’ ones for which $\alpha_i \geq 0$, the equality now being allowed by our assumption above about constant sizes. In [8], for Kasner type solutions with no energy momentum tensor, such transformations are shown to always exist. Here, for the configurations given in Table I, we see that these transformations exist also in the presence of energy momentum tensor. These transformations can be easily obtained, the ‘safe’ configurations now being 2255, W555, 222, 25W in M theory and FW in string theory.

(iv) Among these ‘safe’ configurations, the configuration 2255 or W555 has maximum entropy for a given energy. See [6] for details. Hence, the universe is likely to be dominated by such configurations. They may, therefore, provide a detailed realisation of the maximum entropic principle that

we have proposed recently in [9] to determine the number $(3 + 1)$ of large spacetime dimensions. Even otherwise, these configurations provide, at the least, a model for our $(3 + 1)$ – dimensional expanding universe.

6. Conclusion

We now briefly summarise the results. We considered N – charge intersecting brane antibrane configurations smeared uniformly in the common transverse space so that they may describe our universe. The brane directions are taken to be toroidal. We found that U dualities imply relations among the scale factors, which then are characterised by N independent functions.

The energy momentum tensor for such a configuration is of the form $T^\mu{}_\nu = \text{diag}(-\rho, p_i)$ which may be determined, in principle, by brane antibrane dynamics. It follows from Einstein’s equations that, as a consequence of U duality, ρ and p_i for $N = 1$ case obey a relation given in equation (13). We then presented an ansatz for $T^\mu{}_\nu$ for the N – charge case which, as can be verified, yields all the U duality relations among the scale factors.

We studied configurations with $q_1 = \dots = q_N$ and $\bar{q}_1 = \dots = \bar{q}_N$ and found, among other things, that the configuration 2255 or W555 leads asymptotically to an effective $(3 + 1)$ – dimensional expanding universe. This result follows as a consequence of U dualities in M theory and is independent of the details of the brane antibrane dynamics.

We studied configurations with $q_1 = \dots = q_N = \bar{q}_1 = \dots = \bar{q}_N$ which, for entropic reasons, are likely to dominate the universe. We assumed that $p_\perp = w\rho$. General solutions to the equations of motion can then be obtained from those given in [5]. We listed the asymptotic solutions for a few N – charge configurations. It is seen, following the reasoning given in [8], that the ‘safe’ configurations are 2255, W555, 222, 25W in M theory and FW in string theory. Among these, the configuration 2255 or W555 has maximum entropy and, hence, is likely to dominate the universe and may, therefore, provide a detailed realisation of the maximum entropic principle proposed in [9]. Even otherwise, these configurations provide, at the least, a model for our $(3 + 1)$ – dimensional expanding universe.

We conclude by mentioning a few issues for further study.

It is important to obtain general solutions to the equations of motion with no restriction on charges. This, however, requires the knowledge of brane antibrane dynamics that determines $\rho(q, \bar{q})$ and $p_\perp(q, \bar{q})$. In the absence of such a knowledge, one may perhaps proceed by making a suitable ansatz for $\rho(q, \bar{q})$ and $p_\perp(q, \bar{q})$.

It may be of interest to obtain the gauge fields $A_{\mu\nu\rho}$ corresponding to such a general ansatz or, atleast, for the simpler ansatz $p_\perp = w\rho$ used in the present paper.

Here, we assumed the brane directions to be toroidal. It may also be of interest to understand the consequences of U dualities for more general

topologies.

Configurations with $q_1 = \dots = q_N = \bar{q}_1 = \dots = \bar{q}_N$ are likely to dominate the universe for entropic reasons. Also, the reasonings given in [8] are invoked here in restricting the relevant configurations to the ‘safe’ ones. Given the importance of these configurations in determining the number $(3+1)$ of large spacetime dimensions, it is crucial to understand how such a condition on charges and such a restriction to ‘safe’ configurations emerge dynamically.

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